Basic Concepts

* Input/Output Ports
  - Connectors where input/output signals enter/leave the circuit

  - Both has Voltage Differences and Current Flows

* Load
  - Elements attached to output port
  - Elements consumes output signal

* Source
  - Supply of input signal as voltage or current
    - Voltage source
    - Current source

  - Ideal Source:
    - Source that does not dissipate its own signal
      - Voltage source w/ zero serial impedance
      - Current source w/ infinite parallel impedance

  - Non-Ideal Source: with finite signal dissipation
    - Voltage source w/ finite voltage drop though finite serial impedance
    - Current source w/ finite current leak though finite parallel impedance

\[ V_2 = V_S - V_Z \]

\[ i_T = i_S - i_Z \]
Thévenin Equivalent Circuits

- Abstraction of electric/electronic circuit into a non-ideal voltage source.
  
- Ideal voltage source:
  \[ U_{TH} = \text{open circuit voltage of the circuit (OC)} \]

- Source (Serial) Impedance:
  \[ Z_{TH} = \text{passive impedance of the circuit with all internal voltage sources replaced by Short Circuit (SC), and all internal current sources replaced by Open Circuit (OC)} \]

Norton Equivalent Circuits

- Abstraction of electric/electronic circuit into a non-ideal current source.

- Ideal current source:
  \[ I_N = \text{Short Circuit current of the circuit (SC)} \]

- Source (Parallel) Impedance:
  \[ Z_N = \text{passive impedance of the circuit with all internal voltage sources replaced by Short Circuit (SC), and all internal current sources replaced by Open Circuit (OC)} \]

Conversion of Equivalent Circuits

- \[ I_N = I_{SC} = \frac{U_{TH}}{Z_{TH}} \]
- \[ U_{TH} = U_{OC} = I_N Z_N \]
- \[ Z_{TH} = Z_N \]
N-MOSTET Transistor i-I Behavior

**Cut-off Region**

\[ U_{GS} < V_T, \quad i_D = 0 \]

**Triode Region**

\[ U_{GS} - V_T \geq U_{BS} \geq 0 \]

\[ i_D = K_n (U_{GS} - V_T) U_{BS} \]

**Saturation Region**

\[ U_{BS} \geq U_{GS} - V_T \geq 0 \]

\[ i_D = \frac{K_n}{2} (U_{GS} - V_T)^2 (1 + \lambda U_{BS}) \]

---

**Threshold Voltage**

\[ V_T = V_{TH} + \gamma (\sqrt{U_{BS} + 2\phi_T} - \sqrt{2\phi_T}) \]

**Parameters**

- \( V_T \): Threshold Voltage
- \( V_{TH} \): Zero-Bias (w.r.t. Body) Threshold Voltage
- \( K_n \): Transconductance
- \( 2\phi_T \): Surface Potential
- \( \lambda \): Channel Length Modulation
- \( \gamma \): Body Effect
DC & AC Analysis of Amplifier Circuit [13.2]

DC Analysis

Step 1: Construct DC Equivalent Circuit by doing the following:
- replace capacitors with open circuits
- replace inductors with short circuits

Step 2: Find Q-point of transistors (in DC Equivalent Circuit) by using large signal model of transistor

AC Analysis

Step 1: Construct AC Equivalent Circuit by doing the following:
- replace capacitors with short circuits
- replace inductors with open circuits
- DC voltage sources with ground connections (AC ground)
- DC current sources with open circuits
**Electronics (II): Single Transistor Amplifiers**

### Biasing [Eq 4.8]

#### Constant $V_{GS}$ Biasing with Voltage Divider

$$V_{Q} = \frac{R_{1}}{R_{1}+R_{2}} \times V_{DD}$$

$$= 3V$$

$$I_D = \frac{K_{n}}{2} (V_{GS} - V_{T})^2 = 50\mu A$$

$$V_{DS} = V_{DD} - I_D R_D = 5V$$

\[V_{G} - V_{T} = 2V\]

\[V_{DS} > V_{G} - V_{T} > 0\] Device is saturated

\[nMOS: \quad V_{T} = 1V \quad K_{n} = 25\, \text{mV/V}^2 \quad \lambda = 0 \quad I_S = 0 \quad I_c = 0\]

#### Four Resistor Biasing

$$V_{Q} = \frac{R_{1}}{R_{1}+R_{2}} \times V_{DD}$$

$$V_{Q} = V_{GS} + V_{RS} = V_{GS} + I_D R_S$$

3rd: $I_D = \frac{K_{n}}{2} (V_{GS} - V_{T})^2$

\[\Rightarrow \quad V_{GS} = V_{Q} - I_D R_S = \left( \frac{R_{1}}{R_{1}+R_{2}} \right) V_{DD} + \frac{K_{n} R_S}{2} (V_{DS} - V_{T})^2\]
Small Signal Model of Transistor Operation

\[ Y_{in} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \]

- \( Y_{in} \): DC input conductance
- \( Y_{12} \): DC transconductance reverse
- \( Y_{21} \): DC transconductance forward
- \( Y_{22} \): DC output conductance

nMOSFET Small Signal Model

\[ G_m = \frac{\partial i_{ds}}{\partial V_{ds}} \]
\[ \left| \begin{array}{c} 5-	ext{pt} \\ \text{gate} \end{array} \right| = K_n (V_{gs} - V_T) (1 + \lambda V_{ds}) = \frac{2I_D}{V_{gs} - V_T} \]

\[ Y_{ds} = \frac{\partial i_{ds}}{\partial V_{ds}} \left| \begin{array}{c} 5-	ext{pt} \\ \text{source} \end{array} \right| = \lambda \frac{K_n}{2} (V_{gs} - V_T)^2 = \frac{\lambda I_D}{V_{gs} - V_T} \]

\[ R_0 = \frac{1}{Y_{22}} = \frac{1 + \lambda V_{ds}}{\lambda I_D} \approx \frac{V_{ds} + \frac{1}{\lambda I_D}}{\lambda I_D} \approx \frac{1}{\lambda I_D} \quad \frac{1}{\lambda I_D} \gg V_{ds} \]

\[ \left| \begin{array}{c} \text{gate} \\ \text{source} \end{array} \right| \]
**ELECTRONICS (I): REVIEW SUMMARY**

**Basic Electronics, Concepts**

**Gain**

Ratio (possibly complex) between input and output signals (in time/frequency/Laplace domains).

- Voltage Gain: $\|A_V\| = \frac{\|V_o\|}{\|V_i\|} = 4A_v$
- Current Gain: $\|A_I\| = \frac{\|I_o\|}{\|I_i\|}$
- Trans-impedance: $\|A_Z\| = \frac{\|V_o\|}{\|I_i\|}$
- Trans-admittance: $\|A_Y\| = \frac{\|I_o\|}{\|V_i\|}$

**Linear Amplifier**

- $V_{in}$
- $I_{in}$
- $V_{out}$
- $I_{out}$

**Power Gain**

$a_p = \frac{\|V_{rms}\|}{\|I_{rms}\|}$

**RMS-Mean-Square Voltage/Current**

DC-equivalent voltage/current values with equivalent power levels.

$V_{rms} = \frac{1}{T} \int_{0}^{T} V(t) \, dt$

For sinusoidal signals $V(t) = V_m \cos(\omega t)$,

$V_{rms} = \frac{V}{\sqrt{2}}$

**Total Harmonic Distortion (THD)**

(Distorted)

Non-ideal Output

$V_{o}(t) = V_0 + V_1 \sin(\omega t + \phi_1) + V_2 \sin(2\omega t + \phi_2) + V_3 \sin(3\omega t + \phi_3) + \ldots$

Decibel Scale

$A_p(dB) = 10 \log\|A_p\|$

$A_V(dB) = 20 \log\|A_V\|$

$A_I(dB) = 20 \log\|A_I\|$

$THD(\%) = 100 \times \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_0}$
ELECTRONICS I - ASSIGNMENT (2):
SINGLE MOSFET TRANSISTOR COMMON-SOURCE (CS) AMPLIFIER

Consider the single MOSFET\(^1\) Transistor Common-Source (CS) Amplifier (with \(V_{dd} = +10V\)) shown in the diagram and try to produce step-by-step the following results:

1. Determine the quiescent point (Q-point) and the operation mode of the MOSFET transistor [30 points]
2. Construct the small-signal model of the MOSFET transistor around its quiescent point [20 points]
3. Draw the AC equivalent circuit of the amplifier around its quiescent point [10 points]
4. Determine the small signal voltage gain of the amplifier [20 points]
5. Determine the small signal input impedance of the amplifier [10 points]
6. Determine the small signal output impedance of the amplifier [10 points]

---

\(^1\) Ignore the polarity of the MOSFET device. It is drawn as a p-MOS device because the n-MOS symbol is absent from the VISIO template.
Given the following nMOSFET CS single-transistor amplifier:

\[ V_{DD} = 10 \text{V} \]
\[ 1.5 \Omega \]
\[ R_i \]
\[ 75 \Omega \]
\[ R_o \]
\[ C_i \]
\[ 200 \mu F \]
\[ 1 \Omega \]
\[ R_2 \]
\[ 50 \mu F \]
\[ V_o \]
\[ C_o \]
\[ 200 \mu F \]

**nMOSFET Device Parameters**

- \( K_n = 40 \mu A/V^2 \)
- \( V_T = 1 \text{V} \)
- \( \lambda = 0.01 \text{V}^{-1} \)

**Part [1]** Determine the Quiescent Point (Q-Point) and the Operation Mode of the transistor.

**Step (1)** We need to construct its DC Equivalent Circuit:

**Step (2)** Solve for \( V_{GS} \) using DC Analysis

\[ V_{GS} = V_{G} - V_{RS} \quad \text{[1]} \]

where

\[ V_{G} = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} \quad \text{[2]} \]

\[ V_{RS} = I_D R_S \quad \text{[3]} \]

Assume nMOSFET operates in its **Saturation Region** (as desired),

then

\[ I_D = \frac{K_n}{2} \left( V_{GS} - V_T \right)^2 \left(1 + \lambda V_{SS} \right) \]

by ignoring the small influence of \( \lambda V_{SS} \) and thus simplify the equation.

we have

\[ I_D \approx \frac{K_n}{2} \left( V_{GS} - V_T \right)^2 \quad \text{[4]} \]
[1] Gently. Substitute eq. [1], [3], [2] into eq. [1], we get

\[ V_{GS} = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} - \frac{K_a R_s}{2} (V_{GS} - V_T)^2 \]

But in numeric values, we have:

\[ V_{GS} = \frac{1}{25} \times 10^4 - \frac{40 \times 10^5 \times 50 \times 10^3}{2} (V_{GS} - 1)^2 \]

\[ = 4 - \frac{2}{7} (V_{GS} - 1)^2 \]

\[ = 4 - (V_{GS}^2 - 2V_{GS} + 1) \]

\[ \Rightarrow V_{GS}^2 - 2V_{GS} + 1 + V_{GS} = 4 \]

\[ \Rightarrow V_{GS}^2 - V_{GS} - 3 = 0 \]

\[ \Rightarrow V_{GS} = \frac{1 \pm \sqrt{1 + 12}}{2} = \frac{1 \pm \sqrt{13}}{2} \]

\[ \Rightarrow V_{GS} = \frac{1 \pm 3.6056}{2} = 2.3028, -1.3028 \]

Reject \( V_{GS} = -1.3028 \) because it causes nMOS to cut off.

Thus, \( V_{GS} = 2.3028 \) V

From eq. [4], we obtain

\[ I_D = \frac{20 \times 10^5}{2} \times (2.3028 - 1)^2 = 33.946 \mu A \]

Thus, \( V_{DS} = V_{DD} - V_{GD} - V_{RS} \)

\[ = V_{DD} - I_D (R_D + R_s) \]

\[ = 10 - 33.946 \times 10^{-6} (75 + 50) \times 10^3 \]

\[ = 10 - 4.2432 \]

\[ = 5.7568 \] V.

nMOS Operating Mode: Since \( V_{DS} = 5.7568 \) V \( > V_{GS} - V_T = 1.3028 \) V

nMOS is operating in its Saturation Region.
Part 2: Construct the Small Signal Model of MOSFET transistor at its Quiescent Point

A MOSFET transistor can be regarded as a transconductance amplifier (with voltage input and current output) when it operates in its saturation region.

The small-signal model is shown below:

Input Impedance $\to \infty$
as the Gate lead is insulated from Drain & Source.

It's defined by two parameters:

**Small Signal Transconductance:**

$$G_m \equiv \frac{\partial I_d}{\partial U_{gs}} \bigg|_{Q-PT} = \frac{2I_d}{V_{gs}-V_T} \left( 1 + \lambda V_{ds} \right) = \frac{2I_d}{V_{gs}-V_T}$$

**Small Signal Output Impedance $R_{ds}$:**

$$\frac{1}{R_{ds}} \equiv \frac{\partial I_d}{\partial U_{ds}} \bigg|_{Q-PT} = \lambda \frac{K_n}{2} (V_{gs}-V_T)^2 = \frac{\lambda I_d}{1 + \lambda V_{ds}},$$

or

$$R_{ds} = \frac{1 + \lambda V_{ds}}{\lambda I_d} \approx \frac{1}{\lambda I_d} \text{ if } \lambda V_{ds} \ll 1.$$
Part [3]

Draw AC Equivalent Circuit.

Please refer to the attached diagrams (a) to (d) for the steps to construct and then simplify the AC Equivalent Circuit.

![Diagram](image)

Figure 13.33 (a) Common-source amplifier circuit employing a MOSFET. (b) ac Equivalent circuit for common-source amplifier in part (a). The common-source connection should now be apparent. (c) ac Equivalent circuit with the MOSFET replaced by its small-signal model. (d) Final equivalent circuit for ac analysis of the common-source amplifier.
Part E(i) Determine Small-Signal Voltage Gain.

Notice: Both resistances of input signal source and output load should be included in the calculation of voltage gain.

We can divide diagram (d) of Part E(i) into input and output stages.

The input stage determines $U_{gs}$:

$$U_{gs} = \left( \frac{R_s}{R_a + R_i} \right) U_i$$

The output stage relates $U_{gs}$ with circuit output $U_o$:

$$U_o = -R_l I_d$$
$$= -R_l g_m U_{gs}$$

where $R_l = R_d || R_o || R_0$.

By substitution, we get:

$$U_o = -g_m \left( \frac{R_s}{R_a || R_i} \right) \left( \frac{R_s}{R_a + R_i} \right) U_i$$

Hence, the voltage gain:

$$A_v = \frac{U_o}{U_i} = -g_m \left( \frac{R_s}{R_a + R_i} \right) \left( \frac{R_s}{R_a || R_i} \right)$$
Part 5] Determine Small-Signal Input Impedance ($R_i$).

By definition, $R_i$ should appear in the AC Equivalent Circuit as shown:

[Diagram]

Comparing the circuit with that in diagram (c), $U_i$.

we realize:

$$R_i = R_i' = R_1 || R_2$$


By definition, $R_o$ should appear in the AC Equivalent Circuit as shown:

[Diagram]

Comparing it with diagram (c),

we deduce:

$$R_o = R_o' = R || R_o$$