Bipolar Junction Transistor (BJT)

- A three-terminal device that uses the voltage of the two terminals to control the current flowing in the third terminal.
  - The basis for amplifier design.
  - The basis for switch design.
  - The basic element of high speed integrated digital and analog circuits.
- Applications
  - Discrete-circuit design.
  - Analog circuits.
    * High frequency application such as radio frequency analog circuit.
  - Digital circuits.
    * High speed digital circuit such as emitter coupled circuit (ECC).
    * Bi-CMOS (Bipolar+CMOS) circuits that combines the advantages of MOSFET and bipolar transistors.
      - MOSFET: high-input impedance and low-power.
      - Bipolar transistors: high-frequency-operation and high-current-driving capabilities.
- Circuit symbol
  - The arrowhead on the emitter implies the polarity of the emitter-base voltage.
    * NPN: $v_{BE} > 0$.
    * PNP: $v_{EB} > 0$.

7.1 Structure

7.1.1 NPN Transistor

- Figure 7.2 depicts a simplified NPN transistor.
  - Emitter (E): heavily doped n-type region.
  - Base (B): lightly doped p-type region.
  - Collector (C): heavily doped n-type region.
  - Two diodes connected in series with opposite directions.
    * EBJ: Emitter-Base junction.
Sec 7.1. Structure

Figure 7.1: Circuit symbols of (a) NPN and (b) PNP transistors.

Figure 7.2: A simplified structure of the NPN transistor.

* CBJ: Collector-Base junction.

- Figure 7.3 shows the cross-section view of an NPN transistor.
  - The NPN transistor has asymmetrical structure.
  - $\alpha$ and $\beta$ parameters are different for forward active and reverse active modes.
- Modes of operations
  - Cutoff
    * EBJ (Reverse), CBJ (Reverse)
    * $v_{BE} < 0$, $v_{CB} > 0$.
  - Active (refer to Figure 7.7)
    * EBJ (Forward), CBJ (Reverse)
    * $v_{BE} > 0$, $v_{CB} > 0$.
  - Reverse Active
    * EBJ (Reverse), CBJ (Forward)
    * $v_{BE} < 0$, $v_{CB} < 0$.
  - Saturation
    * EBJ (Forward), CBJ (Forward)
    * $v_{BE} < 0$, $v_{CB} < 0$. 

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Lecture 7. Bipolar Junction Transistor (BJT)

Figure 7.3: Cross-section of an NPN BJT.

- Figure 7.4 shows the voltage polarities and current flow in the NPN transistor biased in the active mode.

Figure 7.4: Voltage polarities and current flow in the NPN transistor biased in the active mode.

7.1.2 PNP Transistor

Figure 7.5: A simplified structure of the PNP transistor.

- Figure 7.5 depicts a simplified PNP transistor.
  - Emitter (E): heavily doped p-type region.
Sec 7.2. Operations of NPN Transistor

- Base (B): lightly doped n-type region.
- Collector (C): heavily doped p-type region.
- Two diodes connected in series with opposite directions.
  * EBJ: Emitter-Base junction.
  * CBJ: Collector-Base junction.

• Modes of operations
  - Cutoff
    * EBJ (Reverse), CBJ (Reverse)
    * $v_{EB} < 0$, $v_{BC} < 0$.
  - Active (refer to Figure 7.7)
    * EBJ (Forward), CBJ (Reverse)
    * $v_{EB} > 0$, $v_{BC} > 0$.
  - Reverse Active
    * EBJ (Reverse), CBJ (Forward)
    * $v_{EB} < 0$, $v_{BC} < 0$.
  - Saturation
    * EBJ (Forward), CBJ (Forward)
    * $v_{EB} > 0$, $v_{CB} > 0$.

• Figure 7.6 shows the voltage polarities and current flow in the PNP transistor biased in the active mode.

![Figure 7.6](image)

*Figure 7.6: Voltage polarities and current flow in the PNP transistor biased in the active mode.*

7.2 Operations of NPN Transistor

7.2.1 Active Mode

• Emitter-Base Junction
Figure 7.7: Current flow in an NPN transistor to operate in the active mode.

- Forward bias, \( v_{BE} > 0 \).
- Electrons in the emitter region are injected into the base causing a current \( i_{E1} \).
- Holes in the base region are injected into the emitter region causing a current \( i_{E2} \).
  - Generally, \( i_{E1} >> i_{E2} \).
  - \( i_E(t) = i_{E1} + i_{E2} \) \hspace{1cm} (7.1)

- Base region
  - Figure 7.8 depicts the concentration of minority carriers (electrons) in the base region.
  - Tapered concentration causes the electrons to diffuse through the base region toward the collector.
    - Some of the electrons may combine with the holes causing a concave shape of the profile.
    - The recombination process is quite small due to lightly doped and thin base region.
      - \( n_p(0) = n_p0e^{v_{BE}/V_T} \) \hspace{1cm} (7.2)
  - Diffusion current \( I_n \) (flowing from right to the left) is proportional to the slope of the concentration profile.
    - \( A_E \) is the cross-sectional area of the base-emitter junction.
    - \( D_n \) is the electron diffusivity in the base region.
    - \( W \) is the effective width of the base.
      - \( I_n = A_EqD_n \frac{dn_p(x)}{dx} = -A_EqD_n \frac{n_p(0)}{W} \) \hspace{1cm} (7.3)
Sec 7.2. Operations of NPN Transistor

- Collector-Base Junction
  - Reverse bias, \( v_{BC} > 0 \).
  - The electrons near the collector side are swept into the collector region causing zero concentration at the collector side.

- Collector current, \( i_C \).
  - Most of the diffusing electrons will reach the collector region, i.e., \( i_C = -I_n \).
    * Only a very small percentage of electrons are recombined with the holes in the base region.
  - As long as \( v_{CB} > 0 \), \( i_C \) is independent of \( v_{CB} \).
    * The electrons that reach the collector side of the base region will be swept into the collector as collector current.

\[
i_C = -I_n = A_E q D_n n_p(0) \frac{1}{W} = A_E q D_n n_{p0} e^{v_{BE}/V_T} = \frac{A_E q D_n n_p^2}{W N_A} e^{v_{BE}/V_T} = I_S e^{v_{BE}/V_T}
\]

- Saturation current (also known as scale current) \( I_S = (A_E q D_n n_p^2) / (W N_A) \)
  * A strong function of temperature.
  * Proportional to the cross-sectional area of the base-emitter junction.
  * Inverse proportional to the base width \( W \).

- Base current \( i_B \)
- $i_B$ is composed of two currents.
  * The holes injected from the base region into the emitter region.
    
    $i_{B1} = \frac{AEqD_n n_i^2}{N_D L_p} e^{v_{BE}/V_T}$  \hspace{1cm} (7.5)
    
  * The holes that have to be supplied by the external circuit due to the recombination.
    
    $\tau_b$ is the average time for a minority electron to recombine with a majority hole.
    
    $i_{B2} = \frac{1}{2} \frac{AEqW n_i^2}{\tau_b N_A} e^{v_{BE}/V_T}$  \hspace{1cm} (7.6)
    
- Formulation of $i_B$ in terms of $i_C$.
  * $I_s$ is the saturation current of $i_C$ (refer to Eq.(7.4))
  * $\beta = 1/ \left( \frac{D_p}{D_n} \frac{N_A W}{N_D L_p} + \frac{1}{2} \frac{W^2}{D_n \tau_b} \right)$ is a constant (normally in the range $50 \sim 200$) for a given transistor.
  * $\beta$ is mainly influenced by (1) the width of the base region, and (2) the relative dopings of the base region and the emitter region $\frac{N_A}{N_D}$.
    
    - To achieve high $\beta$ values, the base should be thin ($W$ small) and lightly doped, and the emitter heavily doped.

\[
\begin{align*}
  i_B &= i_{B1} + i_{B2} \\
  &= I_s \left( \frac{D_p}{D_n} \frac{N_A W}{N_D L_p} + \frac{1}{2} \frac{W^2}{D_n \tau_b} \right) e^{v_{BE}/V_T} \\
  &= \left( \frac{D_p}{D_n} \frac{N_A W}{N_D L_p} + \frac{1}{2} \frac{W^2}{D_n \tau_b} \right) i_C \\
  &= \frac{1}{\beta} \times i_C \\
\end{align*}
\]  \hspace{1cm} (7.7)

- Emitter current $i_E$
  - From KCL, the $i_E$ and $i_C$ can be related as follows:

\[
\begin{align*}
  i_E &= i_B + i_C \\
  &= \frac{1}{\beta} i_C + i_C \\
  &= \frac{1 + \beta}{\beta} \times i_C \\
  &= \frac{1}{\alpha} \times i_C \\
  &= \frac{1}{\alpha} \times I_s e^{v_{BE}/V_T} \\
\end{align*}
\]  \hspace{1cm} (7.8)

* $\alpha = \beta / (1 + \beta) \simeq 1$ is a constant for a given transistor.
Sec 7.2. Operations of NPN Transistor

* Small change in $\alpha$ corresponds to large changes in $\beta$.

** Recapitulation
- Configuration
  * EBJ (Forward), CBJ (Reverse)
- Relationship between $i_C$, $i_B$, and $i_E$.
  * $i_C = \beta \times i_B$.
  * $\beta$ (normally in the range 50~200) is a constant for a given transistor.
  * $i_C = \alpha \times i_E$.
  * $\alpha (\beta / (1 + \beta) \lesssim 1)$ is a constant for a given transistor.
- $i_B$, $i_C$, and $i_E$ are all controlled by $v_{BE}$.

$$
\begin{align*}
  i_C &= I_S e^{v_{BE} / V_T} \\
  i_B &= \frac{1}{\beta} I_S e^{v_{BE} / V_T} \\
  i_E &= \frac{1}{\alpha} I_S e^{v_{BE} / V_T}
\end{align*}
$$

- Figure 7.9 depicts the large signal equivalent model of the NPN transistor.
  * In Figure 7.9 (a), $i_C$ behaves as a voltage ($v_{BE}$) controlled current source.
    $$i_C + i_B = i_E = \frac{1}{\alpha} i_C$$
  * In Figure 7.9 (b), $i_C$ behaves as a current ($i_E$) controlled current source.
    $$i_C + i_B = i_E$$
    $$\Rightarrow \alpha i_E + i_B = i_E$$

* The diode $D_E$ represents the forward base-emitter junction.

### 7.2.2 Reverse Active Mode
- The $\alpha$ and $\beta$ in the reverse active mode are much lower than those in the forward active mode.
  - $\alpha_R$ is in the range of 0.01 to 0.5.
  * In forward active mode, the collector virtually surrounds the emitter region.
    - Electrons injected into the thin base region are mostly captured by the collector.
  * In reverse active mode, the emitter virtually surrounds the collector region.
    - Electrons injected into the thin base region are partly captured by the collector.
7.2.3 Ebers-Moll (EM) Model

- A composite model that can be used to predict the operations of the BJT in all possible modes.
  - Combine Figure 7.9 (b) and Figure 7.10.
- $\alpha$ and $\beta$
Sec 7.2. Operations of NPN Transistor

Figure 7.11: Ebers-Moll model of the NPN transistor.

- $\alpha_F$ and $\beta_F$ denotes the parameters in forward active mode.
- $\alpha_R$ and $\beta_R$ denotes the parameters in reverse active mode.

**Equivalent saturation current $I_{SE}$ and $I_{SC}$**

- From Figure 7.9 (b) and Figure 7.10, $I_{SE}$ and $I_{SC}$ are the equivalent saturation currents at the EBJ and CBJ, respectively.

\[
I_{SE} = \frac{1}{\alpha_F} I_S \\
I_{SC} = \frac{1}{\alpha_R} I_S \\
\Rightarrow \alpha_F I_{SE} = \alpha_R I_{SC} = I_S
\]  

(7.12)

**$i_C$, $i_B$, and $i_E$ in the EM model**

\[
i_E = i_{DE} - \alpha_R i_{DC} \\
i_C = -i_{DC} + \alpha_F i_{DE} \\
i_B = (1 - \alpha_F) i_{DE} + (1 - \alpha_R) i_{DC}
\]  

(7.13)

- $i_{DE} = I_{SE} \left(e^{v_{BE}/V_T} - 1\right)$.
- $i_{DC} = I_{SC} \left(e^{v_{BC}/V_T} - 1\right)$. 

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• By Eq. (7.12),

\[
\begin{align*}
    i_E &= \frac{I_S}{\alpha_F} \left( e^{v_{BE}/V_T} - 1 \right) - I_S \left( e^{v_{BC}/V_T} - 1 \right) \\
    i_C &= I_S \left( e^{v_{BE}/V_T} - 1 \right) - \frac{I_S}{\alpha_R} \left( e^{v_{BC}/V_T} - 1 \right) \\
    i_B &= \frac{I_S}{\beta_F} \left( e^{v_{BE}/V_T} - 1 \right) + \frac{I_S}{\beta_R} \left( e^{v_{BC}/V_T} - 1 \right)
\end{align*}
\]

(7.14)

- \( \beta_F = \alpha_F / (1 - \alpha_F) \).
- \( \beta_R = \alpha_R / (1 - \alpha_R) \).

### 7.2.4 Saturation Mode

• CBJ is in forward bias, i.e., \( v_{BC} > 0.4V \).
  - CBJ has larger junction area than EBJ.
    * CBJ has larger saturation current \( I_S \) and lower cut-in voltage than EBJ.
    * In forward bias,
      * The voltage drop across CBJ is 0.4V.
      * The voltage drop across EBJ is 0.7V.
  - As \( v_{BC} \) is increased, \( i_C \) will be decreased and eventually reach zero.

\[
i_C \simeq I_S e^{v_{BE}/V_T} - \frac{I_S}{\alpha_R} e^{v_{BC}/V_T}
\]

(7.15)

**Figure 7.12:** Concentration profile of the minority carriers in the base region of an NPN transistor.
Figure 7.13: Current flow in a PNP transistor biased to operate in the active mode.

### 7.3 Operations of PNP Transistor

#### 7.3.1 Active Mode

- Current in a PNP transistor is mainly conducted by holes.
- Emitter-Base Junction
  - Forward bias, $v_{EB} > 0$.
  - Holes in the emitter region are injected into the base causing a current $i_{E1}$.
  - Electrons in the base region are injected into the emitter region causing a current $i_{E2}$.
    - Generally, $i_{E1} \gg i_{E2}$.

$$i_E(t) = i_{E1} + i_{E2} \quad (7.16)$$

- Base region
  - Tapered concentration causes the holes to diffuse through the base region toward the collector.
    - Some of the holes may combine with the electrons.
    - The recombination process is quite small due to lightly doped and thin base region.
- Collector-Base Junction
  - Reverse bias, $v_{BC} > 0$.
  - The holes near the collector side are swept into the collector region causing zero concentration at the collector side.
- Collector current, $i_C$.
  - Most of the diffusing holes will reach collector region.
    - Only a very small percentage of holes are recombined with the electrons
in the base region.

- As long as $v_{BC} > 0$, $i_C$ is independent of $v_{BC}$.
  * The holes that reach the collector side of the base region will be swept into
  the collector as collector current.

- Base current $i_B$
  - $i_B$ is composed of two currents.
    * The electrons injected from the base region into the emitter region.
    * The electrons that have to be supplied by the external circuit due to the
      recombination.

- Emitter current $i_E$
  - From KCL, the $i_E$ and $i_C$ can be related as follows:

$$i_E = i_B + i_C$$
$$= \frac{1}{\beta}i_C + i_C$$
$$= \frac{1 + \beta}{\beta} \times i_C$$
$$= \frac{1}{\alpha} \times i_C$$
$$= \frac{1}{\alpha} \times I_S e^{v_{EB}/V_T}$$

  * $\alpha = \beta / (1 + \beta) \simeq 1$ is a constant for a given transistor.
  * Small change in $\alpha$ corresponds to large changes in $\beta$.

- Figure 7.14 depicts the large signal equivalent model of the PNP transistor.
7.3.2 Reverse Active Mode

- Similar to NPN transistor.

7.3.3 Saturation Mode

- Similar to NPN transistor.

7.3.4 Summary of the $i_C$, $i_B$, $i_E$ Relationships in Active Mode

- NPN transistor

\[
\begin{align*}
   i_C &= I_s e^{v_{BE}/V_T} \\
   i_B &= \frac{I_s}{\beta} e^{v_{BE}/V_T} \\
   i_E &= \frac{I_s}{\alpha} e^{v_{BE}/V_T}
\end{align*}
\]
Figure 7.16: The $i_C - v_{CB}$ characteristics of an NPN transistor.

$$
\begin{align*}
    i_C &= \alpha i_E \\
    i_C &= \beta i_B \\
    i_B &= (1 - \alpha)i_E = \frac{i_E}{1 + \beta} \\
    i_E &= (1 + \beta)i_B 
\end{align*}
$$

- PNP transistor.
  - The $v_{BE}$ in Eq. (7.18) is replaced by $v_{EB}$.

### 7.4 The $i - v$ Characteristics of NPN Transistor

#### 7.4.1 Common Base ($i_C - v_{CB}$)

- Figure 7.16 depicts the $i_C$ versus $v_{CB}$ for various $i_E$, which is also known as the common-base characteristics.
  - Input port: emitter and base terminals.
    - Input current $i_E$.
  - Output port: collector and base terminals.
    - Output current $i_C$.
  - The base terminal serves as a common terminal to both input port and output port.
- Active Region ($v_{CB} \geq -0.4V$)
  - $i_C$ depends slightly on $v_{CB}$ and shows a small positive slope.
Sec 7.4. The $i - v$ Characteristics of NPN Transistor

- $i_C$ shows a rapid increase, known as breakdown phenomenon, for a relatively large value of $v_{CB}$.
- Each $i_C - v_{CB}$ curve intersects the vertical axis at a current level equal to $\alpha I_E$.
  * Total or large-signal $\alpha$ (common-base current gain)
    \[ \alpha = \frac{i_C}{i_E}, \text{ where } i_C \text{ and } i_E \text{ denote the total collector and emitter currents, respectively.} \]
  * Incremental or small-signal $\alpha$
    \[ \alpha = \frac{\Delta i_C}{\Delta i_E}. \]
  * Usually, the values of incremental and total $\alpha$ differ slightly.

- Saturation Region ($v_{CB} < -0.4V$)
  - CBJ is forward biased.
  - The EM model can be used to determine the $v_{CB}$ at which $i_C$ is zero.

7.4.2 Common Emitter ($i_C - v_{CE}$)

- Figure 7.17 depicts the $i_C$ versus $v_{CE}$ for various $v_{BE}$, which is also known as the common-emitter characteristics.
  - Input port: base and emitter terminals.
    * Input current $i_B$.
  - Output port: collector and emitter terminals.
    * Output current $i_C$.
    - The emitter terminal serves as a common terminal to both input port and output port.

- Active Region ($v_{CB} \geq -0.4V$)
  - $i_C$ increases as the $v_{CE}$ is increased, which is known as Early Effect.
    * At a given $v_{BE}$, increasing $v_{CE}$ increases the width of the depletion region of the CBJ.
    * The effective base width $W$ is decreased.
    * As shown in Eq. (7.4), $I_S$ is inversely proportional to the base width $W$.
  - When extrapolated, the characteristics line meet at point on the negative $v_{CE}$ (normally in the range of 50V to 100V), $-V_A$.
    * $V_A$ is a constant for a given transistor.

- Large signal equivalent circuit model in active mode.
  - The linear dependency of $i_C$ on $v_{CE}$ can be formulated as follows:
    \[ i_C = I_S e^{v_{BE}/V_A} (1 + \frac{v_{CE}}{V_A}) = I_C (1 + \frac{v_{CE}}{V_A}) \quad (7.20) \]
  - The output resistance looking into the collector-emitter terminals.
    * Inversely proportional to the collector current $I_C$ without considering Early effect.
Figure 7.17: The $i_C - v_{CE}$ characteristics of the BJT.

* Controlled by $v_{BE}$.

$$\Delta i_C = I_S e^{v_{BE}/V_T} \left( \frac{\Delta v_{CE}}{V_A} \right)$$  \hspace{1cm} (7.21)

$$\Rightarrow r_o = \frac{\Delta v_{CE}}{\Delta i_C} = \frac{V_A}{I_C}$$

Figure 7.18 depicts the large signal equivalent circuit model of an NPN BJT in the active mode and with the common emitter configuration.

* Figure 7.18 (a), voltage $v_{BE}$ controls the collector current source.

* Figure 7.18 (b), the base current $i_B$ controls the collector current source $\beta \times i_B$.

- Large signal or DC $\beta$

  * The ratio of total current in the collector to the total current in the base, which represents the ideal current gain (where $r_o$ is not present) of the common-emitter configuration.

$$\beta_{dc} = \frac{i_C}{i_B} \bigg|_{v_{CE}=\text{constant}}$$ \hspace{1cm} (7.22)

  * $\beta$ is also known as the common-emitter current gain.

- Incremental or AC $\beta$

  * Short-circuit common-emitter current gain.

  * AC $\beta$ and DC $\beta$ differ approximately 10% to 20%.

$$\beta_{ac} = \frac{\Delta i_C}{\Delta i_B} \bigg|_{v_{CE}=\text{constant}}$$ \hspace{1cm} (7.23)
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Figure 7.18: Large signal equivalent circuit model of an NPN BJT operating in the active mode and with common-emitter configuration.

Figure 7.19: An expanded view of the common-emitter characteristic in the saturation region.

- Saturation Region ($v_{CB} < -0.4V$)
  - Figure 7.19 depicts an expanded view of the common-emitter characteristic in the saturation region.
  - Analytical expressions of $i_C - v_{CE}$ using EM model.
    * $v_{BE} = v_{CE} + v_{CB}$.

\[
\begin{align*}
  i_C & \simeq I_S (e^{v_{BE}/V_T}) - \frac{I_S}{\alpha_R} (e^{v_{BC}/V_T}) \\
  I_B & \simeq \frac{I_S}{\beta_F} (e^{v_{BE}/V_T}) + \frac{I_S}{\beta_R} (e^{v_{BC}/V_T}) \\
  i_C & \simeq (\beta_F I_B) \left( \frac{e^{v_{CE}/V_T} - \frac{1}{\alpha_R}}{e^{v_{CE}/V_T} - \frac{2}{\beta_F \beta_R}} \right)
\end{align*}
\]
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Figure 7.20: Plot of normalized $i_C$ versus $v_{CE}$ for an NPN transistor with $\beta_F = 100$ and $\alpha_R = 0.1$.

- Large signal equivalent circuit model in saturation mode.
  - The saturation transistor exhibits a low collector-to-emitter resistance $R_{CEsat}$.
    
    $$R_{CEsat} = \left. \frac{\partial v_{CE}}{\partial i_C} \right|_{i_B=i_B,i_C=I_C} \approx \frac{1}{10\beta_F I_B} \quad (7.26)$$

  - At the collector side, the transistor is modeled as a resistance $R_{CEsat}$ in series with a battery $v_{CEoff}$ as shown in Figure 7.21 (c).
    * $V_{CEoff}$ is typically around 0.1V.
    * $V_{CEsat}$ is typically around 0.1 $\sim$ 0.3V.

    $$V_{CEsat} = V_{CEoff} + I_{Csat} R_{CEsat} \quad (7.27)$$

  - For many applications, the even simpler model shown in Figure 7.21 is used.
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Figure 7.21: Equivalent circuit representation of the saturated transistor.