3 Additional Notes on Frequency Response

\[ \frac{\mathcal{V}_S(s)}{\mathcal{V}_{\text{in}}(s)} \rightarrow \text{Amp} \rightarrow \mathcal{V}_0(s) \]

\[ A_v(s) = \frac{\mathcal{V}_0(s)}{\mathcal{V}_{\text{in}}(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n} \]

* We are most interested in the values of \( A_{\text{mid}}, \omega_L, \omega_H \)

* Poles and zeros can be separated into two groups.
  
  - Those associated with the low frequency response below the mid-band can be combined into a function \( \mathcal{H}_L(s) \).

\[ \mathcal{H}_L(s) = \frac{(s+w_{21}^L)(s+w_{22}^L)(s+w_{23}^L)\cdots(s+w_{2k}^L)}{(s+w_{11}^L)(s+w_{12}^L)(s+w_{13}^L)\cdots(s+w_{1k}^L)} \]

\[ |\mathcal{H}_L(j\omega)| \rightarrow 1 \quad \text{for} \quad \omega \gg \omega_{2j}^L, \omega_{1j}^L, j=1\ldots k \]
Those associated with the high frequency response above the mid-band region can be grouped into a function $F_H(s)$:

$$F_H(s) = \frac{(1 + \frac{s}{\omega_{b1}^H})(1 + \frac{s}{\omega_{b2}^H}) \ldots (1 + \frac{s}{\omega_{bz}^H})}{(1 + \frac{s}{\omega_{p1}^H})(1 + \frac{s}{\omega_{p2}^H}) \ldots (1 + \frac{s}{\omega_{pz}^H})}$$

$$|F_H(j\omega)| \rightarrow 1 \text{ for } \omega < \omega_{b1}^H, \omega_{p1}^H, i = 1 \ldots l$$

* $A_H(s) = A_{mid} \cdot F_c(s) \cdot F_H(s)$

- At low frequencies, $A_H(s) \approx A_{mid} \cdot F_c(s)$

$$20 \log |A_H(s)| = 20 \log |A_{mid}| + 20 \log |F_c(s)|$$

- **D**ominate **L**ow-**F**requency **P**ole, $\omega_L = \omega_p$

- Zeros are at frequencies low enough to not influence the lower-cutoff frequency $\omega_L$. 
One of the poles is much larger than the others.

\[ H(s) \approx \frac{s}{s+W_p} \]

Given that we focus on the frequencies around the \( W_p \).

\[ |H(j\omega)| = \frac{(\omega^2 + W_{z1}^2)^{\frac{1}{2}}(\omega^2 + W_{z2}^2)^{\frac{1}{2}} \ldots (\omega^2 + W_{zK}^2)^{\frac{1}{2}}}{(\omega^2 + W_{p1}^2)^{\frac{1}{2}}(\omega^2 + W_{p2}^2)^{\frac{1}{2}} \ldots (\omega^2 + W_{pk}^2)^{\frac{1}{2}}} \]

\( W \gg W_{z1}, \ldots, W_{zK} \) \quad \text{Dominant Pole at Low Freq.}

\( W \gg W_{p1}, \ldots, W_{pk} \) \quad \text{except } W_p.

\[ |H(j\omega)| = \left( \frac{\omega^2}{\omega^2 + W_p^2} \right)^{\frac{1}{2}} \Rightarrow H(j\omega) = \frac{j\omega}{j\omega + W_p} \]

\[ \Rightarrow H(s) = \frac{s}{s + W_p} \]

- When there is no dominant pole, use \( \text{sTC} \).

\[ W_s = \sum_{i=1}^{n} \frac{1}{R_i \times C_i} \]

- \( R_i \) represents the resistance at terminals of the \( i \)th capacitor \( C_i \) with all the other capacitors replaced by short circuits.

- \( R_sC_i \) is the short-circuit-time-constant (SCTC) associated with \( C_i \).
- At high frequencies, \( A_H(s) \approx A_{\text{mid}} \cdot H_H(s) \)

\[ 20 \log |A_H(s)| = 20 \log_{10} (A_{\text{mid}}) + 20 \log_{10} (H_H(s)) \]

- Dominant high frequency pole, \( H_H \approx W_p \)

\[ \rightarrow \text{Zeros are at infinite frequencies or high enough} \]
\[ \text{to not influence the value of } H_H(s) \text{ near } W_H. \]

\[ \rightarrow \text{One of the poles is much smaller than the others.} \]

\[ H_H(s) \approx \frac{1}{i \cdot \frac{s}{W_p}} \rightarrow \text{Dominant Pole at high frequency.} \]

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When there is no dominate pole, use OCTC for estimating \( W_H \).

\[ W_H \approx \frac{1}{\min \{ \text{RtoCl} \}} \]
- $R_0$ represents the resistance measured at the terminals of capacitor $C_i$ with the other capacitor open circuits.

- $R_0 C_i$ is the open-circuit-time-constant associated with capacitance $C_i$.

$\hat{\text{Example:}}$ Consider the following circuits, find out $A_{mid}$, $\omega_r$, and $\omega_H$ using SCTC and OCTC.

*Small Signal Analysis for determining $A_{mid}$*
- Replace MOST2T with T Model

\[
\begin{align*}
U_g &= \Delta i \cdot \frac{R_G}{R_s + R_G} \\
\Delta d &= \frac{U_g}{g_m} = g_m U_g \\
\Delta V_o &= -\Delta d \cdot (R_o/RL) \\
\Delta V_o &= -(R_o/RL) \cdot g_m \cdot \frac{R_G}{R_s + R_G} \Delta i \\
\frac{\Delta V_o}{\Delta i} &= \text{Amid} = \frac{g_m (R_o/RL) R_G}{R_s + R_G}
\end{align*}
\]

* SCTC for determining \( \sum_c \approx \sum \frac{1}{R_s C_i} \)
\[ R_{s1} = R_s + R_G \]
\[ R_{s2} = R_s + R_0/\gamma_0 \approx R_s + R_0 \]
\[ R_{s3} = R_4/\gamma_m \]

* OECT for determining \[ Wt \approx \frac{1}{\gamma_0 R_0 G} \]

High frequency Small Signal Model

\[ R_s \]
\[ C_{gd0} \]
\[ C_{gs} \]
\[ U_i \]
\[ R_G \]
\[ V_{gs} \]
\[ R_0 (gs) = R_s / R_G \]
\[ R_0 (gd) = \frac{U_i}{i_x} \]
Another view of $R_0 (gd)$

\[
\begin{align*}
\dot{I}_x &= \frac{U}{R_s / R_g} \\
U_x &= U_1 + (I_x + q_m U_1) R_c' \\
U_x &= I_x (R_s / R_g) + (I_x + q_m I_x (R_s / R_g)) R_c' \\
\Rightarrow \frac{\dot{X}_x}{I_x} &= \frac{R_s / R_g}{1 + q_m (R_s / R_g)} R_c' \\
&= R_c' + q_m R_c \\
W_H &= \frac{1}{R_0 (g_s) C_{gs} + R_0 (g_d) C_{gd}}.
\end{align*}
\]
§ Clarification of Ground (gnd)

* It is meaningless to specify the voltage of a node without giving the reference point.

* Ground designates a node in a circuit to be the reference point. The voltages of the other nodes in the same circuit are defined as the difference in potential as compared to the ground node.

* Essentially, all the nodes connected to the ground can be deemed as they are connected to the same node.

* Examples: Consider the following circuit, the ground nodes can be A, B, C, or D. The voltages of the other nodes depend on which node is specified as ground.

![Circuit Diagram]

* In the CS, CD, CG amplifier designs, we normally specify which node of the MOSFET is connected to the ground.
Misunderstanding of the KCL at the ground node.

- The ground does not draw a current out of the loop. It also does not introduce a current into the loop.

- An alternative view of the above circuit is shown as follows.