Lecture 1 Human Vision

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What is Light?
- EM wave characterized by *spectral power distribution*
- Visible EM waves are of wavelength from 380 to 750nm

What is Color?
- Human sensation (color response) in response to light (color stimulus)
- Rays are not colored!
## EM Wave

<table>
<thead>
<tr>
<th>Wave Type</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM</td>
<td>1 km, 1000 m</td>
</tr>
<tr>
<td>TV</td>
<td>10 m</td>
</tr>
<tr>
<td>FM</td>
<td>1 m, 100 cm</td>
</tr>
<tr>
<td>RADAR</td>
<td>10 cm</td>
</tr>
<tr>
<td>MICROWAVES</td>
<td>1 000 000 nm, 0.1 cm, 1 mm</td>
</tr>
<tr>
<td>INFRARED RADIATION</td>
<td>10 000 nm</td>
</tr>
<tr>
<td>VISIBLE RADIATION</td>
<td>1 000 nm</td>
</tr>
<tr>
<td>ULTRAVIOLET RADIATION</td>
<td>100 nm</td>
</tr>
<tr>
<td>X-RAYS</td>
<td>1 nm</td>
</tr>
<tr>
<td>GAMMA RAYS</td>
<td>0.001 nm</td>
</tr>
</tbody>
</table>
Human Vision

- Photoreceptors

  - Photopic Conditions: **Cones** perceive **Color Tone**
  - Scotopic Conditions: **Rods** extract only **Luminance**
Cone and Rod Anatomy

- Light-absorbing pigments initiate vision
Trichromacy

- S-, M-, L-Cones
  - Short (S), Middle (M), and Long (L) wavelengths
  - Trichromatic color theory foundation
- Cone Sensitivity
  - Function of light absorption rate
Metamerism

- Metamerism
  - Different wavelength compositions may evoke same color

- Metameric Set
  - Wavelength compositions showing same color appearance
  - Number of stimuli in a metameric set varies widely
Metamerism Example

- Human CANNOT Identify Wavelength Compositions

by Jeff Beall, Adam Doppelt and John F. Hughes
(c) 1995 Brown University and the NSF Graphics and Visualization Center
Trichromatic Theory of Color Mixture

- Additive Mixture of Three Primary Colors
  - Trichromacy of Human vision\(^1\)
    \[ t = \sum_{k=1,2,3} e_k p_k \]
    
    where
    - \( p_k \): Spectrum of primary colors (discrete version)
    - \( e_k \): Tristimulus values (Intensities/Amounts of primaries)

- Primary Colors
  - Linearly Independent - any two cannot produce the third
  - Choice is arbitrary

\(^1\)Continuous/Accurate Version \( t(\lambda) = \sum_k e_k p_k(\lambda) \)
Color Mixture

- **Additive Color Mixture**
  - Illuminating lights
  - RGB Primary

- **Subtractive Color Mixture**
  - Reflecting lights
  - CMY Primary
Color Wheel

Blue - Magenta
Cyan - Red
Green - Yellow

Complements
Color Matching Experiment

- **Tristimulus $e_k$**
  - Additive mixture CANNOT produce all colors
  - Ex: $t = -|e_1|p_1 + e_2p_2 + e_3p_3$, given $e_1 < 0$ and $e_2, e_3 > 0$

$$t + |e_1|p_1 = e_2p_2 + e_3p_3$$

$TestLight$ $AdditivePrimaryMixture$
Grassmann’s Additivity Law

- **Stimulus**
  \[ \mathbf{t} = [t_1, t_2, t_3, \ldots, t_n]^T \]

- **Primary Color Intensities**
  \[ \mathbf{e} = [e_1, e_2, e_3]^T \]

- **Empirical Linearity**
  - \( \mathbf{t}_a \rightarrow \mathbf{e}_a, \mathbf{t}_b \rightarrow \mathbf{e}_b \)
  - If \( \mathbf{t}_{a+b} = a \times \mathbf{t}_a + b \times \mathbf{t}_b \)
  - Then \( \mathbf{e}_{a+b} = a \times \mathbf{e}_a + b \times \mathbf{e}_b \)

\(^a\)t is discrete version of light spectral power distribution \( t(\lambda) \)
Linearity of Color Matching

- Stimulus \( t \) v.s. Response \( e \)

\[
\begin{bmatrix}
e_1 \\
e_2 \\
e_3
\end{bmatrix} = \begin{bmatrix}
t_1 \\
t_2 \\
t_3 \\
\vdots \\
t_n
\end{bmatrix} \tag{1}
\]

\( \text{System Matrix} \)

- Column Vectors - Tristimulus of Monochromatic Lights
  - Given

\[
t_1 = [1, 0, 0, ..., 0]^T
\]

- Then

\[
e = [e_1, e_2, e_3]^T = [C_{11}, C_{21}, C_{31}]^T
\]
Color Matching Functions

- Rows Vectors - Color Matching Functions (CMF)
  - Extract intensities of primary colors

\[
\begin{bmatrix}
  e_1 \\
  e_2 \\
  e_3
\end{bmatrix} =
\begin{bmatrix}
  \text{CMF of Primary 1}, C_1 \\
  \text{CMF of Primary 2}, C_2 \\
  \text{CMF of Primary 3}, C_3
\end{bmatrix}
\begin{bmatrix}
  t_1 \\
  t_2 \\
  t_3 \\
  . \\
  . \\
  t_n
\end{bmatrix}
\]

where \( C_k = [C_{k,1}, C_{k,2}, ..., C_{k,n}], k = 1, 2, 3 \)

- Tristimulus of Arbitrary Light \( \mathbf{t} \)

\[ e_k = C_k \mathbf{t} \text{ or } e_k = \int C_k(\lambda) t(\lambda) d\lambda, k = 1, 2, 3 \]

- Metamers Under Photopic Conditions

\[ \mathbf{Ct} = \mathbf{Ct'} \]
CMF Example

- Primary Lights
  - 645.2nm (R), 525.3nm (G), 444.4nm (B)

![RGB Color Matching Functions](image-url)
Color Space Conversion

- Color Space Conversion \((e, P, C) \leftrightarrow (e', P', C')\)
  - Primary Spectra: \(P = [p_1, p_2, p_3], P' = [p'_1, p'_2, p'_3]\)
  - Tristimulus Values: \(e = [e_1, e_2, e_3]^T, e' = [e'_1, e'_2, e'_3]^T\)
- Coordinate Conversion \(e \leftrightarrow e'\)
  - Test Light Match \(t\)
    
    \[ e = Ct = C \left[ e_1 p_1 + e_2 p_2 + e_3 p_3 \right] = C \left[ e'_1 p'_1 + e'_2 p'_2 + e'_3 p'_3 \right] \]

    \[ \begin{cases} 
    e = CPe \\
    CPe = CP'e' \Rightarrow e = CP'e' \end{cases} \]

- Color Matching Functions Conversion \(C \leftrightarrow C'\)
  
  \[ \begin{cases} 
  Ct = CP'e' \\
  e' = C't \Rightarrow C' = CP'C \end{cases} \]

- CMF are unique up to linear transformation
Biological Basis of Color Matching

- Cone absorption matrix \( \mathbf{B} \) is system matrix for color matching

\[
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = \mathbf{B} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\
. \\
. \\
. \\
. \\
t_n
\end{bmatrix}
\]

Absorption Function of \( L \) Photopigment
Absorption Function of \( M \) Photopigment
Absorption Function of \( S \) Photopigment

System Matrix
Biological Basis of Color Matching

- $C = QB$ must hold
Human Color Sensation

- **Brightness**
  - Sum of S, M, L Cone responses
  
  \[ Y = \int C(\lambda) a_y(\lambda) \, d\lambda = \int C(\lambda) \left( \sum_i a_i(\lambda) \right) \, d\lambda, \quad i = 1, 2, 3 \]

  where
  - \( a_i(\lambda) \): *Relative Absorption Functions* of Cones (Similar to CMF)
  - \( a_y(\lambda) = \sum a_i(\lambda) \): *Luminous Efficiency Function*

- **Hue**
  - Dominant color - color tone of peak wavelength

- **Saturation**
  - Color purity in terms of spectrum spreadness
Brightness, Hue, Saturation
Scotopic Matching Experiment

- Light Match between Test \( t \) and Primary \( p \) Under Scotopic Conditions
  - Adjust primary intensity \( e \)
  - Primary spectrum \( p \) is arbitrary

- Observations
  - Any \( t \) can be matched by \( e \) \( p \)
  - Linear relationship between \( e \) and \( t \)

\[
e = R_t = \begin{bmatrix} r_1, r_2, \ldots, r_n \end{bmatrix}_\text{SystemMatrix} \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix}
\]

- Metamers Under Scotopic Conditions

\[
R_t = R_t'
\]
Scotopic Sensitivity Function

- Matrix $\mathbf{R} = [r_1, r_2, \ldots, r_n]$ - Scotopic Sensitivity Function
  - Extract intensity coefficient of primary light
- Coordinate Conversion $e \leftrightarrow e'$
  \[
  e = \mathbf{R}t = \mathbf{R}e'\mathbf{p} = \mathbf{R}e'\mathbf{p'} \Rightarrow e = (\mathbf{R}p/\mathbf{R}p') e'
  \]
- Scotopic Sensitivity Function Conversion $\mathbf{R} \leftrightarrow \mathbf{R}'$
  \[
  e = \mathbf{R}t = \mathbf{R}p' e' \text{ and } e' = \mathbf{R}'t \Rightarrow \mathbf{R} = (\mathbf{R}p') \mathbf{R}'
  \]
- $\mathbf{R}$ are unique up to scaling
Biological Basis of Scotopic Matching

- Rod absorption function $A$ is system matrix for Scotopic Matching
  - $A = kR$ must hold
## References

1. Brian A. Wandell - *Foundations of Vision*
2. Yao Wang, et. al - *Video Processing and Communications*
3. G. A. Agoston - *Color Theory and Its Application in Art and Design*