Lecture Notes on Basic Electronics for Students in Computer Science

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Preamble

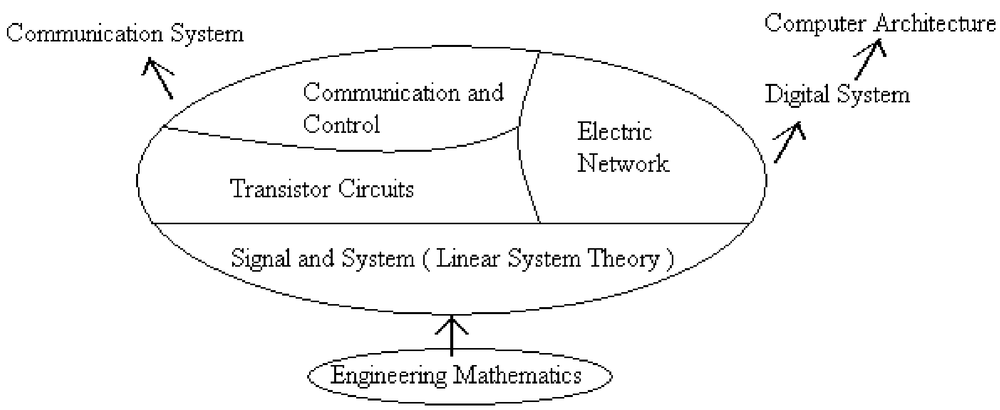
1.1 Goal of This Course

- Analysis and design of electronic circuits.

1.2 Content of This Course

- Electric circuit (Passive Circuit) analysis.
  - Only containing Resistor (R), Capacitor (C), and Inductor (L).
  - No signal amplification.
- Electronic circuit (Active Circuit) analysis.
  - Containing transistor.
  - Single transistor circuit.
  - Signal amplification.
- Operational amplifier and application.

1.3 Relationship with Other Disciplines



Part I

System and Circuit Analysis

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Signals and Systems

2.1 Basic Concepts

Table 2.1: Signals and Systems in Time and Frequency Domains.

	Signals	Systems
Time Domain	Waveforms, $x(t)$	Impulse Response, $h(t)$
Frequency Domain	Spectrum, $X(\omega)$	Frequency Response, $H(\omega)$

Definition 2.1 *System stands for the transformation of signal from one to another. It can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way, resulting in other signals as outputs.*

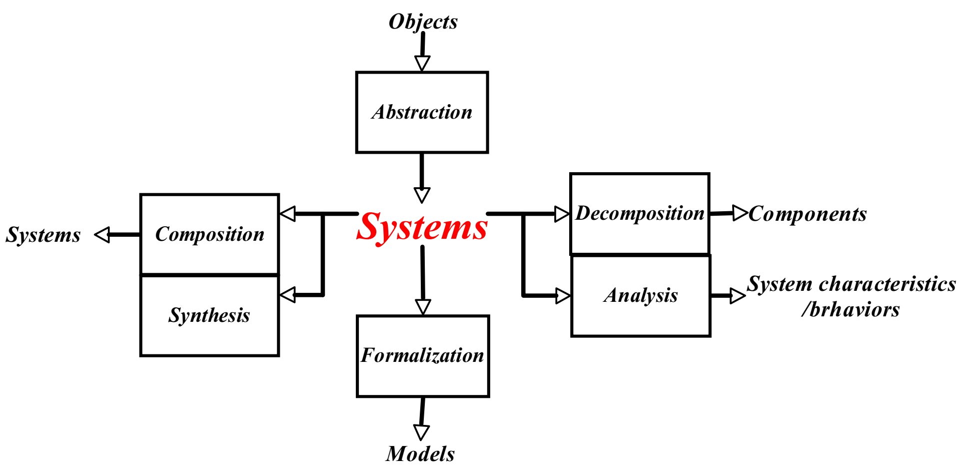
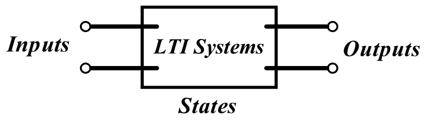


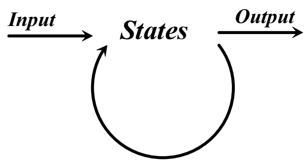
Figure 2.1: System from engineering perspective.

- Approach
  - Abstraction
    - Representing a real object by its special characteristics; that is, the relation between its inputs and outputs, which becomes a system.

  - Decomposition
    * Dividing a system into several smaller systems (components) and studying them to understand the large system.
  - Composition
    * Putting several systems together to form a larger system and studying it.
- Model



- Relative



- Perspectives
  - Time domain.
  - Frequency domain.

2.2 Types of Systems

- **Temporal characteristic**
  - Continuous-Time Systems
    * Inputs/outputs of the systems are functions defined at continuous time.

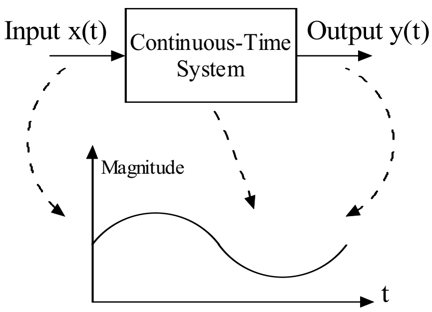


Figure 2.2: Continuous-time systems.

  - Discrete-Time Systems

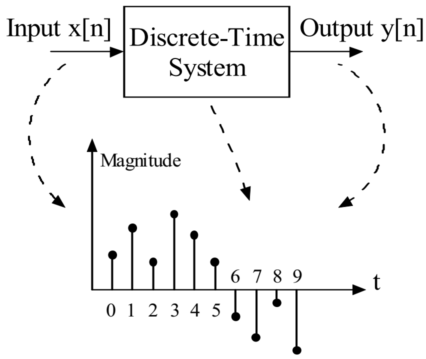


Figure 2.3: Discrete-time systems.

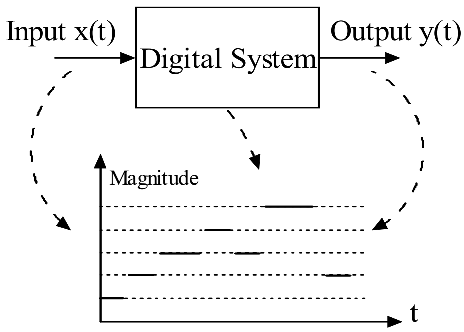


Figure 2.4: Digital system.

        * Inputs/outputs of the systems are functions defined at discrete time.

- **Magnitude**
    - Analog systems
        * Inputs/outputs of the system have continuously varying values.
    - Digital systems
        * Inputs/outputs of the system have discrete values.

2.3 Axiomatic Properties of Systems

- **Linearity**
    - If an input consists of the weighted sum of several signals, then the output is the superposition of the responses of the system to each of those signals.

$$\begin{aligned} y1(t) &= H\{x1(t)\} \\ y2(t) &= H\{x2(t)\} \\ a \times y1(t) + b \times y2(t) &= H\{a \times x1(t) + b \times x2(t)\} \end{aligned} \tag{2.1}$$

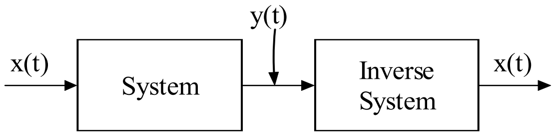


Figure 2.5: Inverse system.

- **Time-Invariant**
  - The behavior and characteristics of the system are fixed over time.
  - For example, the magnitudes of resistors and capacitors of a circuit are unchanged over time.

$$\begin{aligned} y(t) &= H\{x(t)\} \\ y(t-\tau) &= H\{x(t-\tau)\} \end{aligned} \quad (2.2)$$

- **Causality**
  - The output of the system depends only on the inputs at the present time and in the past.
  - For example, $y(t) = \int_0^t x(\tau)d\tau$.
- **Invertability**
  - Distinct inputs of the system lead to distinct outputs, and an inverse system exists.
  - For example, a system which is $y(t) = 2x(t)$, for which the inverse system is $y(t) = \frac{1}{2}x(t)$.
- **Stability**
  - If the input of the system is bounded, then the output must be bounded.

2.4 Time-Domain Analysis

Definition 2.2 *The analysis of a LTI system that is based on the relationship between time-varying inputs and their corresponding time-varying outputs.*

- Inputs/outputs are time functions (waveforms).

2.4.1 Impulse Response

- Output of the system with a fictitious input of Direc-Delta function $\delta(t)$.
  - For LTI systems, the system characteristic in time domain is the system impulse response.

- Direc-Delta function $\delta(t)$.

$$\delta(t) = \begin{cases} 0 & if \quad t \neq 0 \\ \infty & if \quad t = 0 \end{cases} . \tag{2.3}$$

$$\int_{-T}^{T} \delta(t)dt = 1, \ \forall T > 0. \tag{2.4}$$

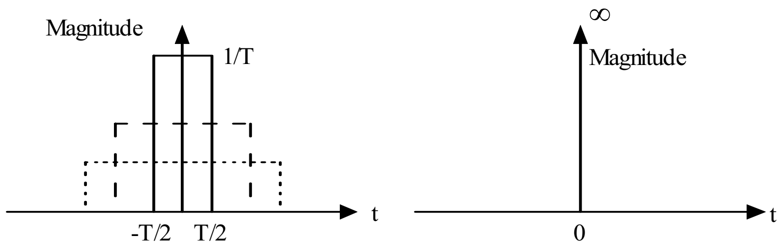


Figure 2.6: Direc-Delta function

  - Signal sampling using Direc-Delta function.

$$x(\tau) = \int_{-\infty}^{\infty} x(t)\delta(t-\tau)dt \tag{2.5}$$

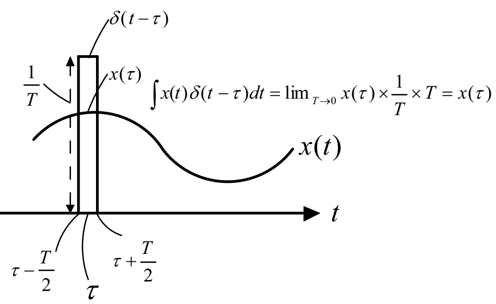


Figure 2.7: Sampling using Direc-Delta function.

  - Signal reconstruction: any real time-functions can be represented by using the integrals of Delta functions as in Eq. (2.6).

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(\tau - t)d\tau \tag{2.6}$$

  - Convolution

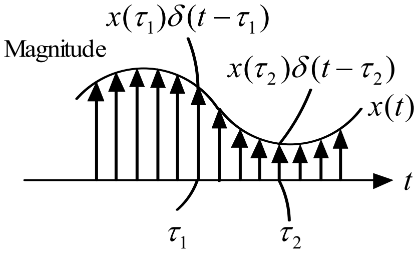


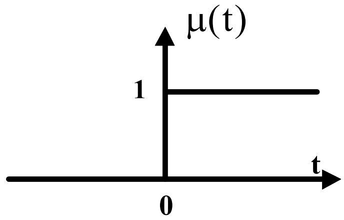
Figure 2.8: A time function represented by a set of delta functions.

* Outputs of the LTI system is the convolution of the input and the system impulse response.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \tag{2.7}$$

2.4.2 Step Response

- Output of the system w.r.t. an input $x(t)$ of step function.



$$x(t) = u(t) = \begin{cases} 0 & if \quad t < 0 \\ 1 & if \quad t \geq 0 \end{cases}. \tag{2.8}$$

$$\frac{du(t)}{dt} = \delta(t). \tag{2.9}$$

2.4.3 Sinusoidal Response

- Output of the system w.r.t. sinusoidal function input $x(t)$.

$$x(t) = \cos(2t) \text{ with } \omega = 2\pi f \tag{2.10}$$

  - * f is frequency.
    * $\omega = 2\pi f$ is the angular frequency.
    * $T = \frac{1}{f}$ is period.
  - For a real LTI system with a sinusoidal input function, the output is also a sinusoidal function but with changes in both magnitude and phase.

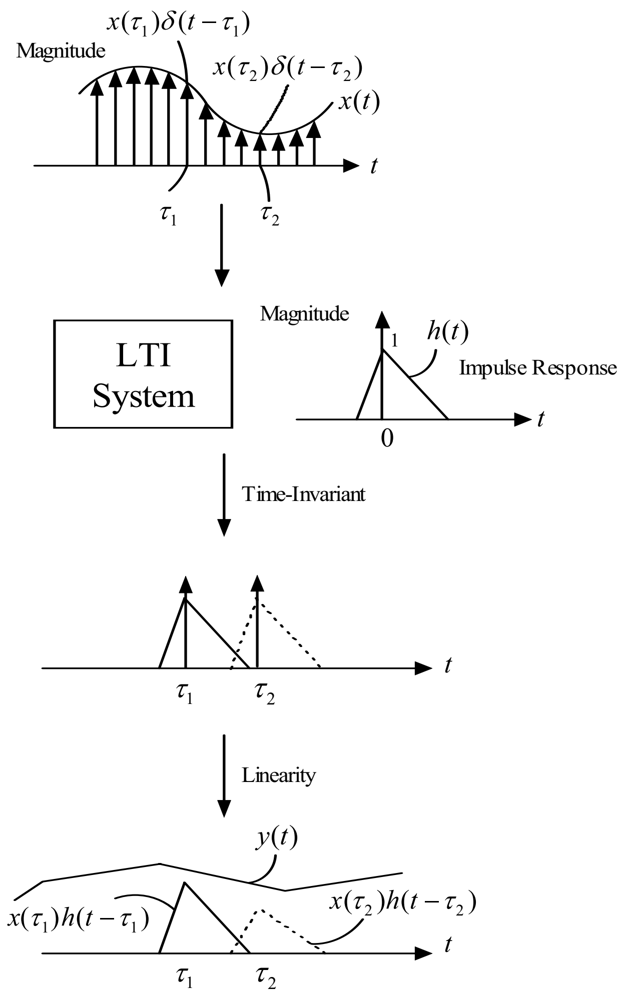


Figure 2.9: Continuous time convolution operation.

$$\begin{aligned} x(t) &= \cos\omega t \stackrel{h(t)}{\rightarrow} y(t) = \|H(\omega)\| \cos\left(\omega t + \measuredangle H(\omega)\right), \\ & \text{where } H(\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau = \|H(\omega)\| \, e^{j\measuredangle H(\omega)} \end{aligned} \tag{2.11}$$

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†**Advanced Topics**

Proof.

$$x(t) = \cos\omega t \stackrel{h(t)}{\rightarrow} y(t) = \|H(\omega)\| \cos\left(\omega t + \measuredangle H(\omega)\right)$$

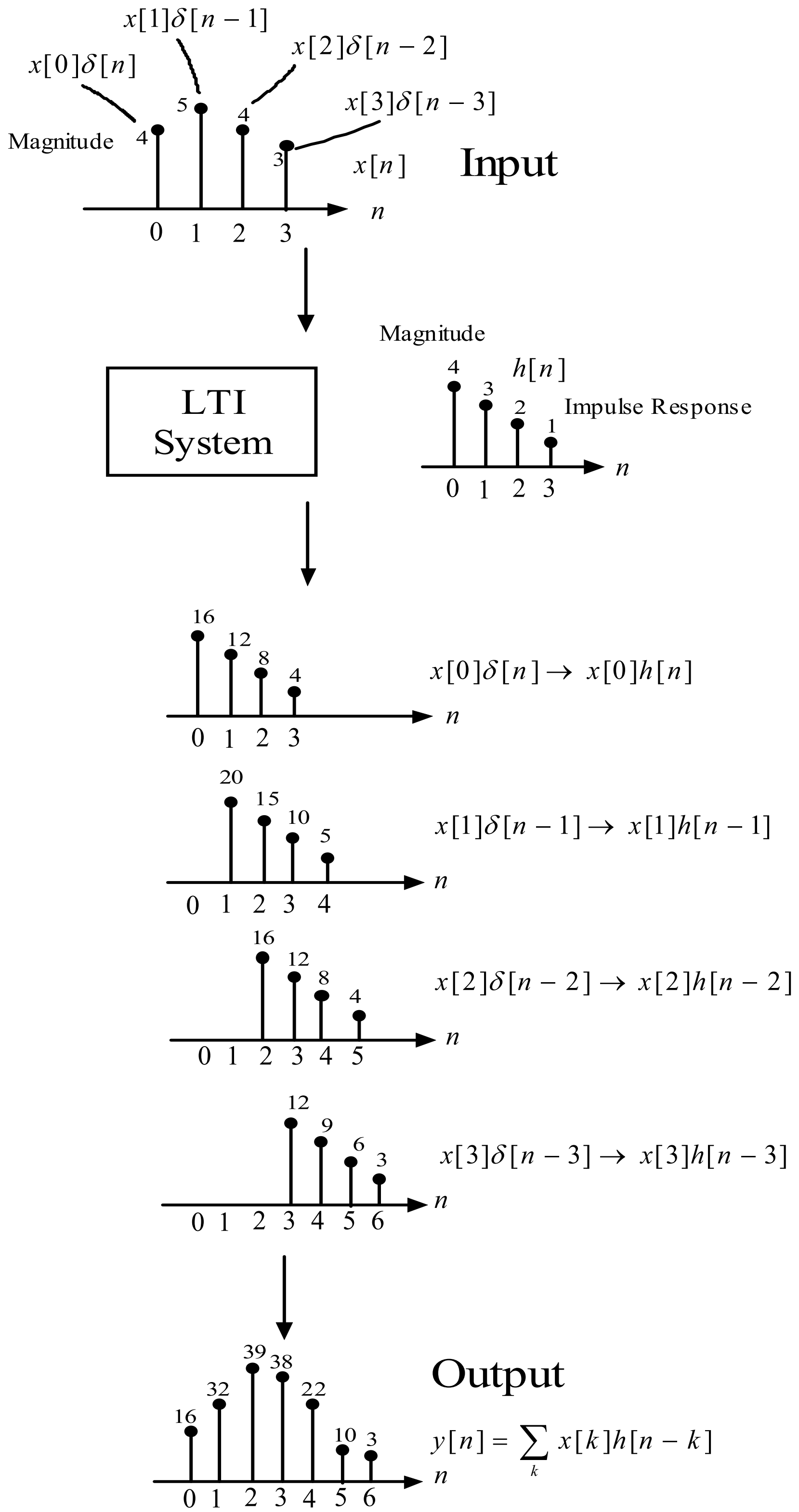


Figure 2.10: Discrete time convolution operation.

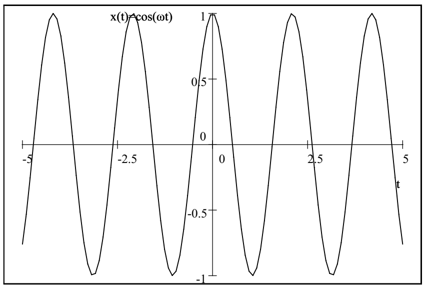


Figure 2.11: Sinusoidal waveform.

$$\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} \cos(\omega\tau)h(t-\tau)d\tau \\
&= \frac{1}{2}\int_{-\infty}^{\infty} (e^{j\omega\tau}+e^{-j\omega\tau})h(t-\tau)d\tau \\
&= \frac{1}{2}\int_{-\infty}^{\infty} e^{j\omega\tau}h(t-\tau)d\tau + \frac{1}{2}\int_{-\infty}^{\infty} e^{-j\omega\tau}h(t-\tau)d\tau
\end{aligned}$$

By defining $\tau^{'} = t-\tau$, the equation above can be written as follows:

$$y(t) = \frac{1}{2}\int_{-\infty}^{\infty} e^{j\omega(t-\tau^{'})}h(\tau^{'})d\tau' + \frac{1}{2}\int_{-\infty}^{\infty} e^{-j\omega(t-\tau')}h(\tau')d\tau'$$

Define $H(\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau^{'}}h(\tau^{'})d\tau' = \|H(\omega)\| e^{j\measuredangle H(\omega)}$ as a complex function of ω. Its complex conjugate is $H^*(\omega) = \int_{-\infty}^{\infty} e^{j\omega\tau^{'}}h^{*}(\tau^{'})d\tau' = \|H(\omega)\| e^{-j\measuredangle H(\omega)}$.Since $h(t)$ is a real function, $h^{*}(\tau^{'}) = h(\tau^{'})$. Thus, $y(t)$ can be formulized as follows:

$$\begin{aligned}
y(t) &= \frac{1}{2}e^{j\omega t}\int_{-\infty}^{\infty} e^{-j\omega\tau^{'}}h(\tau^{'})d\tau' + \frac{1}{2}e^{-j\omega t}\int_{-\infty}^{\infty} e^{j\omega\tau'}h(\tau')d\tau' \\
&= \frac{1}{2}e^{j\omega t} \times \|H(\omega)\| e^{j\measuredangle H(\omega)} + \frac{1}{2}e^{-j\omega t} \times \|H(\omega)\| e^{-j\measuredangle H(\omega)} \\
&= \|H(\omega)\| \cos(\omega t + \measuredangle H(\omega))
\end{aligned}$$

■

- More generally, it is the output of the system w.r.t. complex exponential function input $x(t)$.

$$x(t) = e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \qquad (2.12)$$

– Complex exponential function $e^{j\omega t}$ is the eigenfunction of any LTI systems.

$$\begin{aligned} x(t) = e^{j\omega t} \overset{h(t)}{\longrightarrow} y(t) &= H(\omega)e^{j\omega t} \\ &= \|H(\omega)\| \cos(\omega t + \measuredangle H(\omega)) + j\,\|H(\omega)\| \sin(\omega t + \measuredangle H(\omega)) \end{aligned} \tag{2.13}$$

$*\ H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt.$

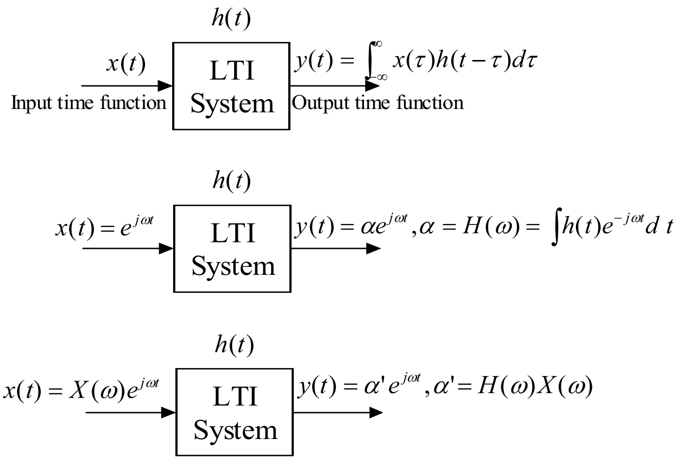


Figure 2.12: The output of a LTI system with exponential complex function.

2.4.4 Initial Value/Driving Free/Natural Response

- Output $y(t)$ w.r.t. null input $x(t) = 0$ and possibly non-zero initial system states. Equivalently, it is solution of Homogeneous System Equation.

2.4.5 Transient Response

- The part of system output that will disappear (die down) as time progress.

$$y_T(t) \longrightarrow 0 \text{ as } t \longrightarrow \infty. \tag{2.14}$$

  – For Linear Time-Invariant (LTI) circuits, $y_T(t)$ = impulse response (with necessary scaling and time-shifting).

2.4.6 Steady-State Response

- The part of system output that will remain after transient response dies down.

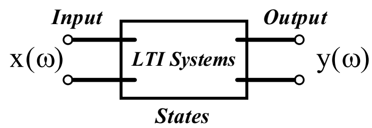
$$y_S(t) = y(t) - y_T(t). \tag{2.15}$$

– For Linear Time-Invariant (LTI) circuits, $y_T(t)$ = impulse response (with necessary scaling and time-shifting).

2.5 Frequency-Domain Analysis

Definition 2.3 *Determination of system output(s) w.r.t complex sinusoidal inputs at different frequencies and with specific initial system state. Results are often displayed along frequency axis or expression as functions of angular frequency (ω).*

- Input X and Output Y are complex functions of angular frequency.

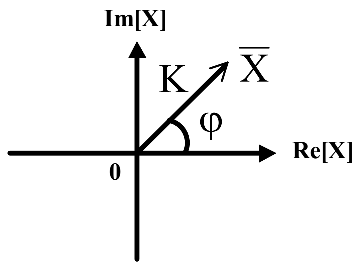


2.5.1 Phasor

- An electrical-engineering representation of sinusoidal signals in frequency domain.
- A constant complex number that encodes the magnitude and the phase of the sinusoidal signals.

Example 2.4 *Given sinusoidal signal $x(t) = K\cos(\omega t + \phi)$, Phasor-$\overline{X} = Ke^{j\phi}$, where $\|\overline{X}\| = K$ and phase $\measuredangle\overline{X} = \phi$.*

- $x(t) = \Re\{\overline{X}e^{j\omega t}\} = K\cos(\omega t + \phi)$.



2.5.2 Spectrum and Fourier Transform

- Any finite-energy signal can be represented by sinusoidal functions (including both sin and cos waveforms) of different frequencies.

– The principle of Inverse Fourier Transform.

$$\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega. \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \|X(\omega)\|\, e^{j\measuredangle X(\omega)}e^{j\omega t}d\omega. \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \{\|X(\omega)\|\cos(\omega t + \measuredangle H(\omega)) + j\,\|X(\omega)\|\sin(\omega t + \measuredangle X(\omega))\}\, d\omega. \end{aligned} \tag{2.16}$$

– $X(\omega)$ is a complex function of angular frequency ω, which expresses the values of the signal phasor in different frequencies.

$$X(\omega) = \|X(\omega)\|\, e^{j\measuredangle X(\omega)}. \tag{2.17}$$

  * $\|X(\omega)\|$ is called magnitude spectrum, specifying the magnitude for different sinusoidal components.
  * $\measuredangle X(\omega)$ is called phase spectrum, specifying the phase for different sinusoidal components.

– $X(\omega)$ can be obtained by taking the Fourier Transform of $x(t)$.

Definition 2.5 *Given $f(t)$, its Fourier Transform, which is defined as follows, is a complex function of the angular frequency ω.*

$$F(\omega) \equiv \Im[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt. \tag{2.18}$$

- Fourier Transform of $f(t)$ is the projection of $f(t)$ on the basis functions $e^{j\omega t}$.

Definition 2.6 *Correspondingly, the Inverse Fourier Transform is as follows:*

$$f(t) \equiv \Im^{-1}[F(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega. \tag{2.19}$$

2.5.3 System Transfer Function $H_s(\omega)$

- A ratio between the spectra of input and output signals of a linear time-invariant circuit.

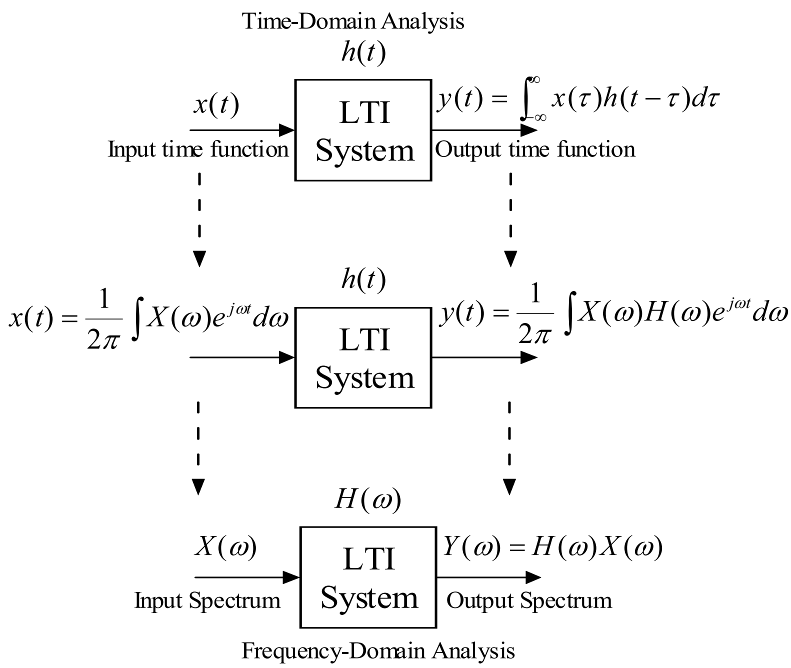


Figure 2.13: Relationship between time-domain analysis and frequency-domain analysis.

– It is also known as the frequency response of a LTI system.

$$\begin{aligned} H_s(\omega) &\equiv \frac{Y(\omega)}{X(\omega)} \\ &= \frac{\|Y(\omega)\|\, e^{j\measuredangle Y(\omega)}}{\|X(\omega)\|\, e^{j\measuredangle X(\omega)}} \\ &= \frac{\|Y(\omega)\|}{\|X(\omega)\|} e^{j(\measuredangle Y(\omega) - \measuredangle X(\omega))} \\ &= \|H_s(\omega)\|\, e^{j\measuredangle H_s(\omega)} \end{aligned} \tag{2.20}$$

∗ $\|H_s(\omega)\| = \|Y(\omega)\| \,/\, \|X(\omega)\|$ is the magnitude response of the system.

∗ $\measuredangle H_s(\omega) = \measuredangle Y(\omega) - \measuredangle X(\omega)$ is the phase response of the system.

- From Eq. (2.13) and Eq. (2.16), each sinusoidal component $X(\omega)e^{j\omega t}$ produces an output signal of $Y(\omega)e^{j\omega t} = X(\omega)H(\omega)e^{j\omega t}$, as shown in Figure 2.13. Thus, $y(t)$ can be written as follows.

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)H(\omega)e^{j\omega t}d\omega. \tag{2.21}$$

- The system transfer function $H_s(\omega) = H(\omega)$, which is the Fourier Transform of the system impulse response $h(t)$.

2.5.4 Time Domain versus Frequency Domain

- The frequency response of a LTI system is the Fourier transform of the system impulse response.

$$H(\omega) = \Im\{h(t)\} \tag{2.22}$$

- The convolution of two signals in time domain is equivalent to the multiplication of their representations in frequency domain.

$$y(t) = x(t) * h(t) \stackrel{\Im}{\longleftrightarrow} H(\omega)X(\omega) \tag{2.23}$$